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Effect of Elasticity on Mixing Torque Requirements for Rushton Turbine Impellers

R. K. PRUD'HOMME and ERIC SHAQFEH

Department of Chemical Engineering
Princeton University
Princeton, NJ 08544

The calculation of torque and power required to mix viscoelastic fluids is an important problem in the design of bulk polymerization reactors, equipment for compounding polymeric fluids, vessels for blending polymer-stabilized latex paints, and fermenters for polysaccharide production. In this note we present a correlation that allows one to assess the effect of fluid elasticity on the torque (or power) required for mixing viscoelastic fluids.

Most work on non-Newtonian mixing is based on papers by Metzner and Otto (1961, 1957) which consider the mixing of a purely-viscous, pseudo-plastic fluid. Their analysis involves obtaining a torque vs. Reynolds number correlation for a vessel from data taken with a Newtonian fluid. An "effective shear rate" in the vessel is determined, and from the viscosity vs. shear rate curve for the non-Newtonian fluid of interest the "effective viscosity" at that "effective shear rate" is determined. A Reynolds number is then calculated using this viscosity and shear rate and this Reynolds number is used in the *Newtonian fluid correlation* to determine the torque. The effects of elasticity are not considered in this analysis, but since elasticity is known to produce differences in flow fields around mixing impellers (White et al., 1977) it is important to known under what conditions torque correlations for viscoelastic fluids differ significantly from those of Newtonian fluids.

An overview of earlier work on the effect of elasticity on mixing non-Newtonian fluids has been presented in the review by Ulbrecht (1974). Previous attempts to draw conclusions about the effect of elasticity on torque have been confounded because most viscoelastic fluids have a strongly shear-thinning viscosity as well as elasticity and, therefore, changes in torque may be due to changes in fluid viscosity or elasticity. It has not been possible to uncouple those two effects. Recently, however, Boger (1977/78) and Walker, Gorell and Homsy (1980) have presented data on a model fluid having a constant viscosity, but having elasticity (primary normal stress difference) that can be varied. This is the fluid we have used. Figure 1 shows the viscosity and primary normal stress difference for three fluids with three levels of polyacrylamide (Dow Chem. Co. Separan AP30) in a solution of corn syrup (CPC International corn syrup #1132) glycerine, and water. A 50/50 solution of glycerine and water was added to the corn syrup to adjust the viscosity to 4.0 Pa · s. Figure 1 shows that the viscosities of the three fluids are constant over the range of shear rate of interest. Without the addition of polyacrylamide the fluid has no measurable primary normal stress. With the addition of 0.015 wt. % polyacrylamide the fluid has significant normal stress and doubling the polyacrylamide concentration (0.03 wt. %) increases the normal stress by an order of magnitude. Using these fluids we can assess the importance of fluid elasticity on mixing torque requirements.

The apparatus used, Figure 2, consists of a 1/4 hp (0.19 kW) variable speed motor, an inductive pick-up tachometer sensing the motion of a 60-tooth sprocket gear, a 0.225 m diameter by 0.263 m tall fully-baffled glass tank, and an air-bearing table with a bonded strain gauge transducer to measure torque. The air-bearing table ensures that there is negligible frictional drag so that low torques can be measured accurately from 0 to 1.2 N · m with an error of less than 0.1% of full scale. Motor speed can be varied from about 20 to 170 rpm. The six-bladed Rushton turbine impeller studied had a 7.5 cm diameter disk with 2.5 cm long by 1.8 cm wide blades attached around the edge.

Figure 3 shows the dimensionless torque versus Reynolds number

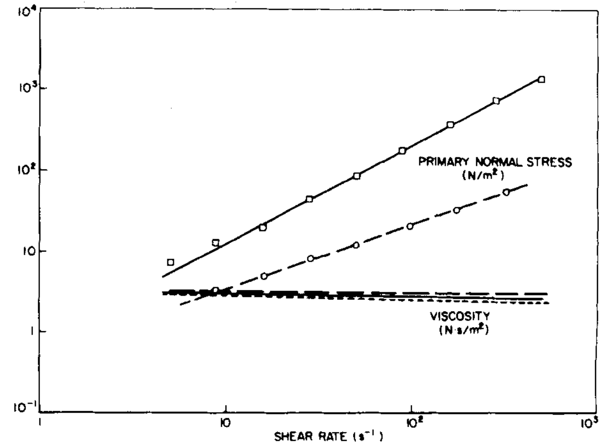

Figure 1. Viscosity and primary normal stress difference for three constant-viscosity, elastic fluids. The fluid with 0% polyacrylamide (△) has no normal stress, the fluid with 0.015% wt. polyacrylamide (○) has the same constant viscosity but significant normal stress, the fluid with 0.03% wt. polyacrylamide (□) has an order of magnitude greater normal stress.

E. Shaqfeh is presently with the Department of Chemical Engineering, Stanford University, Stanford, CA.

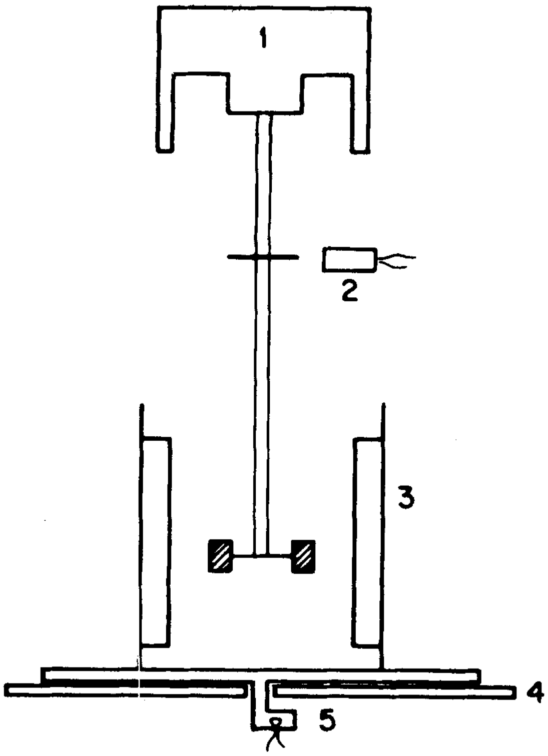

Figure 2. Experimental apparatus.
(1) 1/4 hp variable speed motor,
(2) digital tachometer,
(3) baffled mixing tank,
(4) air-bearing table, and
(5) strain gage torque transducer.

data for mixing the three fluids. The effect of fluid elasticity is substantial—four times the torque is required to mix the most elastic fluid compared to the Newtonian fluid with the same viscosity. To make the torque and elastic forces dimensionless we have chosen a scaling suggested by the analysis by Thomas and Walters (1964) for flow of a second-order elastic fluid around a spinning sphere. Their analysis shows that secondary flows are a result of competition between elastic forces and inertia forces with the result that torque is scaled with viscosity $\Gamma \equiv T\rho/\eta^2R$ and the elastic forces are scaled with inertia $m \equiv N_1/(2\rho R^2\omega^2)$, where Γ is the dimensionless torque, T is the torque, N_1 is the primary normal stress difference, ρ is the density, η is the viscosity, R is the characteristic impeller radius, and ω is the angular frequency of rotation. This elastic parameter m, has been named the Aberystwyth number (Thomas and Walters, 1964). These scalings are in contrast to the usual scaling of torque with inertia forces (drag coefficient $C_D \equiv T/\rho\omega^2D^5$) and elasticity with viscous forces (Weissenberg number $\equiv N_1/\tau_{12}$, where τ_{12} is the shear stress), but we feel that the physics governing the creation of elastically-driven secondary flows in laminar flow favors our scaling. The data have been extrapolated through the origin because there must be no torque at zero Reynolds number. In contrast, the usual drag coefficient, C_D, becomes infinite at zero Reynolds number, making extrapolations difficult.

When these data are reduced to provide a torque correlation for mixing elastic fluids we obtain:

$$\Gamma = (1 + m^{1/4})(12.7Re + 2.41 \times 10^{-3}Re^3), \tag{1}$$

$$\begin{bmatrix}\text{Elastic}\\ \text{Effects}\end{bmatrix}\begin{bmatrix}\text{Newtonian Fluid Correlation}\\ \Gamma_N\end{bmatrix}$$

where the Reynolds number is defined, $Re = \rho\omega R^2/\eta$. We have

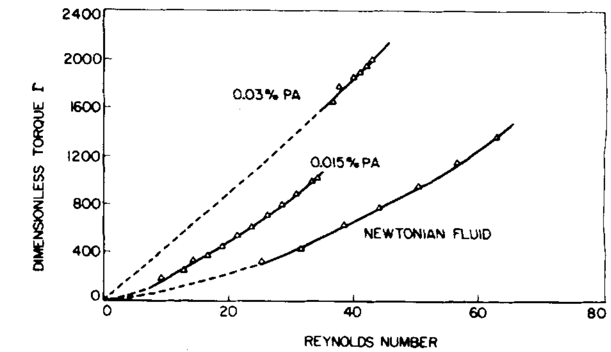

Figure 3. Dimensionless torque vs. Reynolds number for three constant-viscosity elastic fluids. The concentrations of polyacrylamide (*PA*) are given on the figure.

factored the torque into two contributions: the torque that would prevail in mixing a Newtonian fluid, times a contribution due to elasticity. This correlation, which is the first correlation available that explicitly includes fluid elasticity, should provide a basis for assessing whether elastic effects are likely to cause significant increases in mixing torque. It should be kept in mind that the correlation in Eq. 1 is based on data for viscous fluids in laminar flow ($Re < 60$). The level of elasticity and impeller speed have been varied but not the tank size or fluid viscosity. A complete set of experiments is underway on tanks of different sizes, and fluids of different viscosities to fully test the correlation. Flow visualization studies of streamlines around impellers of various configurations in viscoelastic fluids (White et al., 1977) show that secondary flows are qualitatively the same for several impeller geometries. Since the energy required to drive these secondary flows is the reason the torque requirements are higher for mixing elastic fluids, it is possible that the factorization in Eq. 1 may be useful for other impeller geometries. However, a test of the validity of this hypothesis is underway. Interesting predictions of the scale-up of these elastic effects to larger sized vessels follow from Eq. 1 and the data in Figure 1; however, further experiments are required here also. What we have conclusively shown is the viscosity alone is not a sufficient correlating parameter for calculations of the torque required to mix viscoelastic fluids.

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